Supplementary PROJ.4 Notes—
Oblique Stereographic Alternative

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This is an alternative Oblique Stereographic where the projection process involves conformal transformation of the geographic coordinates to the sphere and then application of the basic spherical form of the Oblique Stereographic transformation.

Forward projection

The first part of the forward projection is getting from the ellipsoid geographic coordinates to coordinates on the conformal sphere.

$$\phi_c = 2\arctan\left[K\tan^C(\pi/4+\phi/2)\left(\frac{1-e\sin\phi}{1+e\sin\phi}\right)^{Ce/2}\right]-\pi/2 \quad (1)$$

$$\lambda_c = C(\lambda-\lambda_0) \quad (2)$$

$$R_c = a\frac{\sqrt{1-e^2}}{1-e^2\sin^2\phi_0} \quad (3)$$

$$C = \sqrt{1+\frac{e^2 cos^4\phi_0}{1-e^2}} \quad (4)$$

$$\chi = \arcsin\left(\frac{\sin\phi_0}{C}\right) \quad (5)$$

$$K = \tan(\chi/2+\pi/4)/\left[\tan^C(\phi_0/2+\pi/4)\left(\frac{1-e\sin\phi_0}{1+e\sin\phi_0}\right)^{Ce/2}\right] \quad (6)$$

The following determines the planar coordinates from the conformal sphere coordinates:

$$x = 2R_c\cos\phi_c\sin(\lambda_c) \quad (7)$$

$$y = 2R_c[\cos\phi_{c0}\sin\phi_c-\sin\phi_{c0}\cos\phi_c\cos(\lambda_c)] \quad (8)$$

where

$$k = 1/[1+\sin\phi_{c0}\sin\phi_c+\cos_{c0}\cos\phi_c\cos(\lambda_c)] \quad (9)$$

Inverse projection

The coordinates on the conformal sphere are obtained from

$$\rho = \sqrt{x^2+y^2} \quad (10)$$

$$c = 2\arctan\left(\frac{\rho}{2R}\right) \quad (11)$$

$$\phi_c = \arcsin(\cos c\sin_{c0}+\frac{y}{\rho}\sin c\cos\phi_{c0}) \quad (12)$$

$$\lambda_c = \arctan\left(\frac{x\sin c}{\rho\cos_{c0}\cos c-y\sin_{c0}\sin c}\right) \quad (13)$$

These values are now converted back to the ellipsoid by:

$$\lambda = \lambda_c/C \quad (14)$$

$$\phi_i = 2\arctan\left[\frac{\tan^{1/C}(\phi'/2+\pi/4)}{K^{1/C}\left(\frac{1-e\sin\phi_{i-1}}{1+e\sin\phi_{i-1}}\right)^{e/2}}\right]-\pi/2 \quad (15)$$

with the initial value of $\phi_{i-1} = \phi_c$ and ϕ_{i-1} iteratively replaced by ϕ_i until $|\phi_i - \phi_{i-1}|$ is less than an acceptable error value.

PROJ.4 usage

Projection usage is `+proj=sterea` with latitude and logitude of projection origin (`+lat_0=` and `+lon_0=`) the main items of interest. Of course, scale factor at the origin (`+k_0`) and false eastings and northings apply. The absolute value of the latitude of origin should be kept less than 90° (polar aspect).